Pollution, abatement and balanced growth

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Pollution, Abatement and Balanced Growth*

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Abstract. The analysis of endogenous growth models with pollution often concentrates on steady state trajectories, under the assumption that the steady state is in some sense stable. In the present note we provide examples showing that this issue should be dealt with carefully. We use the Rebelo "Ak" model augmented with a stock of pollutants causing a negative externality. It is found that optimal growth is not necessarily balanced (contrary to the outcome of the standard Rebelo model). Moreover, the existence of the externality may affect long run optimal growth rates.

Key words. Endogenous growth, pollution.

1. Introduction

One of the main questions economists face today is whether economic growth is compatible or can be reconciled with care for the environment. This question has been addressed in numerous articles. Initially, the well-known Ramsey model served as the main tool of analysis. It was found by, among others, Forster (1973), Keeler *et al.* (1972) and, more recently, by van der Ploeg and Withagen (1991) that, in the long run, aggregate capital and consumption are smaller in an economy which has preferences involving nature than in the modified golden rule prevailing in an economy without care for the environment. It was also found that environmental care does not affect growth rates in classical models of economic growth, and it just has a level effect. Hence, growth rates differing between countries cannot be attributed to different attitudes towards the environment. This observation can be considered as an additional drawback of traditional growth models and it is therefore not surprising that recently much interest has emerged for endogenous growth in the spirit of Romer (1986), Lucas (1988), Barro (1990) and Rebelo (1991). We refer to Bovenberg and Smulders (1993), Hofkes (1993), Gradus and Smulders (1993), van der Ploeg, van Marrewijk and Verbeek (1993), Michel and Rotillon (1993), Smulders and Gradus (1993), Vellinga (1993). The vast majority of these papers consider pollution as a flow that is damaging to social welfare. In such a situation it can be shown that long run growth rates are still unaffected. An exception is van der Ploeg *et al.* (1993), who incorporate stock pollution as well in the Barro/Rebelo model. Gradus and Smulders

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(op. cit.) even claim (without proof) that with stock pollution the results with respect to growth remain unaltered. Another common feature of the studies mentioned is the emphasis on balanced growth. All papers study optimal growth and pay special attention to the analysis of steady states or to steady growth. Some of the authors (e.g. Hofkes (1993) and van der Ploeg *et al.* (1993)) derive conditions on the functional forms of functions involved in order to guarantee the existence of balanced growth trajectories as solutions to the optimal control problem. However, to our knowledge none of the authors proves the stability of such trajectories.

The aim of the present paper is twofold. We want to show first that stock externalities can affect optimal growth rates. Secondly, we show that in some cases it is indeed worthwhile to have a look at balanced growth. But in other circumstances balanced growth cannot be optimal, be it that convergence to balanced growth may occur. In order to make these points we obviously do not need a general model. Instead we shall employ a rudimentary form of the Rebelo model with a linear technology, thereby not doing enough justice to the richness of the original underlying economic reasoning.

2. The Standard Rebelo Model

We shall use the following model as a benchmark for the analysis. Society's objective is to maximize social welfare, given by a utilitarian welfare functional, where we choose instantaneous utility to be logarithmic.

$$W = \int_0^\infty e^{-\rho t} \ln C(t) dt. \tag{2.1}$$

Here ρ is the constant and positive rate of time preference. The advantage of this specification is of course that zero rates of consumption can always be discarded as candidates for optimality. Net investments ($\dot{K}$) equal production (aK) minus consumption (C) and depreciation ($\mu_k K$)

$$\dot{K}(t) = (a - \mu_k)K(t) - C(t), \quad K(0) = K_0 \text{ given.}$$

We define $\varphi := a - \rho - \mu_k$. Then the above equation can be written as (omitting the time argument t if there is no danger of confusion):

$$\dot{K} = (\varphi + \rho)K - C, \quad K(0) = K_0 \text{ given.} \tag{2.2}$$

It is easily seen that along an optimal trajectory

$$K(t) = K_0 e^{\varphi t}, \quad C(t) = \rho K(t). \tag{2.3}$$

So, we have balanced growth with growth rate φ.

3. Stock Pollution; No Abatement

One way to model the negative impact of the stock of pollutants (S) on social welfare is to extend (2.1) as follows:

$$W = \int_0^\infty e^{-\rho t}[\ln C(t) - \tfrac{1}{2}S^2(t)]dt. \tag{3.1}$$

We have chosen here for separability of the instantaneous utility function for mathematical convenience. This particular form is the easiest to illustrate our points. An alternative way of modelling the negative impact of pollutants is through the technology (see e.g. Gottinger, 1993). Since we do not introduce abatement as yet, (2.2) remains unaltered. The accumulation of pollutants is described as

$$\dot{S} = bK - \mu_s S, \tag{3.2}$$

where it is assumed that pollution is proportional to production and the regeneration process is exponential ($\mu_s > 0$).

The Hamiltonian corresponding to the problem of maximizing W subject to (2.2) and (3.2) reads

$$\begin{aligned}\mathcal{H}(K, S, C, \bar{\lambda}, \bar{\psi}) := e^{-\rho t}[\ln C - \tfrac{1}{2}S^2] + \bar{\lambda}[(\varphi + \rho)K - C] \\ + \bar{\psi}[bK - \mu_s S).\end{aligned}$$

Defining λ and ψ by $\lambda = \bar{\lambda}e^{\rho t}$ and $\psi = -\bar{\psi}e^{\rho t}$, we can easily derive the following system of necessary conditions.

$$\dot{K} = (\varphi + \rho)K - \frac{1}{\lambda}, \tag{3.3}$$

$$\dot{S} = bK - \mu_s S, \tag{3.4}$$

$$\dot{\lambda} = -\varphi\lambda + b\psi, \tag{3.5}$$

$$\dot{\psi} = (\rho + \mu_s)\psi - S, \tag{3.6}$$

where it is to be understood that $\lambda = 1/C$.

The Jacobian of this system is given by

$$J = \begin{pmatrix} \varphi + \rho & 0 & 1/\lambda^2 & 0 \\ b & -\mu_s & 0 & 0 \\ 0 & 0 & -\varphi & b \\ 0 & -1 & 0 & \rho + \mu_s \end{pmatrix}$$

If $\varphi < 0$, then $\lambda \to \infty$ as $t \to \infty$, so that $C \to 0$ as $t \to \infty$. Furthermore $K \to 0$ and $S \to 0$ because it cannot be optimal to maintain a positive stock of capital while consumption tends to zero.

The interesting case of course the one where $\varphi > 0$. We have balanced positive growth when pollution does not play a role. This will no longer be the case. We show that it is optimal for the state variables K and S to converge to positive constants; also the rate of consumption will converge to a positive constant.

Let $(\hat{K}, \hat{S}, \hat{\lambda}, \hat{\psi})$ solve (3.3)–(3.6) with the left hand side of each of these equations set equal to zero. It follows that

$$\frac{1}{\hat{\lambda}^2} = \frac{(\rho + \mu_s)\mu_s(\rho + \varphi)\varphi}{b^2} .$$

It is then straightforward to prove that the characteristic polynomial reads

$$\begin{aligned} 0 = y^4 - 2\rho y^3 + (\rho^2 - \varphi^2 - \eta^2 - \rho(\varphi + \eta))y^2 \\ + \rho(\varphi^2 + \eta^2 + \rho(\varphi + \eta))y + 2(\eta + \rho)\ (\varphi + \rho)\eta\varphi, \end{aligned}$$

where $\eta = -\mu_s - \rho$. It was shown in van Marrewijk, de Vries and Withagen (1992) that this equation has real solutions with two being negative and two being positive. This implies that the steady state is indeed locally asymptotically stable. (The usual caveat with respect to the non-degeneracy of the stable manifold can be dealt with along the lines set out by Huijberts and Withagen (1992).) So, we find that the asymptotic long run growth rate is zero in this model, whereas it would be positive (for $\varphi > 0$) if there is no concern for the environment. The presence of stock pollution affects the growth rate and does not just have a level effect. Strictly speaking this conclusion holds only locally. But it is easily seen from the differential equations that the stock of capital will not have a positive or negative constant asymptotic growth rate.

Economically this result implies that the possibility of endogenous growth is by itself not sufficient to remove the burden pollution puts on the economy. This is of course due to the absence of an abatement technology, which will henceforth be introduced in the next section.

4. Negative Stock Effects; Abatement

There are many ways to introduce abatement. We shall consider abatement as an activity requiring an amount A of the composite commodity to reduce net emissions. The equation describing the accumulation of capital is now modified to

$$\dot{K} = (\varphi + \rho)K - C - A. \qquad (4.1)$$

With regard to the reduction of net emissions two specifications will be studied. The first one is the following

$$\dot{S} = be^{-A}K - \mu_s S. \qquad (4.2)$$

When $A = 0$ this equation boils down to the one employed in the previous section. The resulting necessary conditions are (4.1) and (4.2) and

$$\dot{\lambda} = -\varphi\lambda + \lambda/K, \qquad (4.3)$$

$$\dot{\psi} = (\rho + \mu_s)\psi - S, \tag{4.4}$$

$$\lambda = 1/C; \quad \dot{\lambda} = \psi b e^{-A} K. \tag{4.5}$$

If $\varphi < 0$ then $\lambda \to \infty$ as $t \to \infty$, so that $C \to 0$ as $t \to \infty$. Furthermore K and S tend to zero.

We shall therefore concentrate on the case $\varphi > 0$. It will be shown that along an optimum $\dot{K}/K \to \varphi$, $\dot{C}/C \to \varphi$ and $A/K \to 0$ as $t \to \infty$. The idea behind the proof is that the efficiency of abatement is so high that it is optimal to forego a tiny bit of consumption initially in order to approach the original growth rate φ later on. In particular, if the stock of capital were bounded, so would pollution and (by (4.4)) the shadow price of pollution (φ) would go to infinity. This implies from (4.5) that consumption must approach zero. Now if capital is bounded from below from zero this cannot be optimal. If capital is arbitrarily close to zero its shadow price displays an unbounded growth rate (by 4.3). The proof proceeds in several steps.

a) Suppose there exists $V > 0$ such that $K(t) \leq V$ for all t. Without loss of generality we can choose V such that $be^{-A(t)}K(t) \leq V$ for all t. Consider

$$\dot{S} = V - \mu_s S, \quad S(0) = S_0.$$

The solution of this differential equation is

$$S(t) = \frac{V}{\mu_s} + \left(S_0 - \frac{V}{\mu_s}\right) e^{-\mu_s t}.$$

There is therefore W > 0 such that $S(t) \leq W$ for all t. Now consider

$$\dot{\psi} = (\rho + \mu_s)\psi - W, \quad \psi(0) = \psi_0.$$

The solution of this differential equation is

$$\psi(t) = -\frac{W}{\rho + \mu_s} + \left(\frac{W}{\rho + \mu_s} + \psi_0\right) e^{(\rho + \mu_s)t}.$$

b) Suppose $K(t) \to 0$ as $t \to \infty$. Then $\dot{\lambda}/\lambda \to \infty$, $\lambda \to \infty$ and $C \to 0$. But $\dot{\lambda}(t) = \psi(t)be^{-A(t)}K(t)$ and $\dot{\psi}/\psi \to \rho + \mu_s$ as $t \to \infty$. This yields a contradiction.

c) Suppose $K(t) \to \bar{K}$ as $t \to \infty$ for some $\bar{K} > 0$. Then we have in view of the results in a) that $\psi \to \infty$, since $\psi(t) > 0$ for all t. We must also have $\dot{C}/C < 0$ eventually in view of (4.5). But it cannot be optimal to have vanishing consumption and a positive stock of capital.

d) Suppose $K(t)$ oscillates around some positive value, X say. Now consider the increasing sequence of points $t_n (n = 1, 2, \ldots)$ for which $K(t_n) = X$ and $\dot{K}(t_n) > 0$. Since $\dot{\lambda}(t_n) = \psi(t_n)be^{-A(t_n)}K(t_n)$ and $A(t_n)$ is bounded from above (because $\dot{K}(t_n) > 0$), we find $\lambda(t_n) \to \infty$ as $n \to \infty$ implying $C(t_n) \to 0$ as $n \to \infty$. But, again, it cannot be optimal not to consume when the stock of capital is positive.

e) It follows from b), c) and d) that for all $V > 0$ and all T there exists $t > T$ such that $K(t) > V$. Suppose K oscillates around some positive M.

If $be^{-A}K$ is bounded from above, we obtain a contradiction as in d). So, $be^{-A}K$ is unbounded, implying that λ/ψ is unbounded. Since C is also unbounded there is a sequence of t's along which $\lambda \to 0$ and $\psi \to 0$. But then also $S \to \infty$ and ψ becomes negative, which is not allowed.

f) We conclude that for all $V > 0$ there exists T such that $K(t) > V$ for all $t > T$. Therefore, in view of (4.2), $\dot{C}/C = -\dot{\lambda}/\lambda \to \varphi$. Moreover, S will be bounded, so that $\dot{\psi}/\psi \to \mu_s + \rho$. It follows from (4.5) that $A/K \to 0$. Q.E.D.

This result shows that balanced growth, defined in the classical way as steadily growing state variables and constant allocative variables (C/K and A/K), is not optimal: there are no constant C/K and A/K satisfying all necessary conditions. On the other hand the economy will *tend* to balanced growth. It is important to note that a linear abatement rule ($A(t) = \alpha t + \beta$ with α large enough) suffices to let pollution go to zero, due to the favourable abatement technology.

Let us now look at a less favourable abatement technology (see also Gradus and Smulders, 1993). The differential equation for pollution is given by

$$\dot{S} = b(K/A)^{\alpha} - \mu_s S, \tag{4.6}$$

where α is a positive constant smaller than unity. In this description a linear abatement program is not sufficient to restrict pollution when capital is growing. As necessary conditions we have (4.1), (4.6) and

$$\dot{\lambda} = -\varphi\lambda + \lambda A/K, \tag{4.7}$$

$$\dot{\psi} = (\rho + \mu_s)\psi - S, \tag{4.8}$$

$$\lambda = 1/C, \quad \lambda = \psi\alpha b(K/A)^{1/\alpha}1/A. \tag{4.9}$$

If we define $x := \lambda K$, then it follows from (4.1), (4.7) and the first part of (4.9) that

$$\dot{x} = \rho x - 1 \tag{4.10}$$

and

$$x(t) = \frac{1}{\rho} + \left(x_0 - \frac{1}{\rho}\right)e^{\rho t}.$$

with x_0 still to be chosen optimally. It follows from (4.9) that

$$x = \psi\alpha b(K/A)^{\alpha+1}.$$

Using this in (4.6) we find

$$\dot{S} = b\left(\frac{x}{\psi\alpha b}\right)^{\frac{\alpha}{1+\alpha}} - \mu_s S. \tag{4.11}$$

Now consider the system of differential equations (4.8), (4.10) and (4.11). Let $(\hat{x}, \hat{\psi}, \hat{S})$ solve the system with the left hand sides set equal to zero. We investigate the stability of the steady state by considering the Jacobian

$$J = \begin{pmatrix} \rho & 0 & 0 \\ v & -\mu_s & w \\ 0 & -1 & \rho + \mu_s \end{pmatrix}$$

with

$$v = \partial\left(b\left(\frac{x}{\psi\alpha b}\right)^{\frac{\alpha}{1+\alpha}}\right)/\partial x,$$

$$w = \partial\left(b\left(\frac{x}{\psi\alpha b}\right)^{\frac{\alpha}{1+\alpha}}\right)/\partial\psi|_{\hat{\psi},\hat{x}} = \frac{\alpha}{1+\alpha}(\rho + \mu_s)\mu_s.$$

The characteristic polynomial is given by

$$(\rho - y)((-\mu_s - y)(\rho + \mu_s - y) + \frac{\alpha}{1+\alpha}(\rho + \mu_s)\mu_s) = 0;$$

$y = \rho$ is one eigenvalue. The other ones are solved from

$$y^2 - \rho y - (\rho + \mu_s)\mu_s \frac{\alpha}{1+\alpha} = 0.$$

Definitely one of these has negative real part, implying that there exists a one-dimensional stable manifold. Therefore, if ψ_0 is optimally chosen on the manifold, we have local asymptotic stability of the steady state. This implies that $C \to \rho K$. Also A/K converges to a constant. Therefore (via (4.7)) $\dot{C}/C$ converges to a constant, which is smaller however than φ. Here we have an example of environmental issues lowering the long run optimal growth rate without reducing it to zero. To the best of my knowledge such an example has not been produced in the literature thus far.

In this subsection we have dealt with two examples of an abatement technology in the Rebelo framework. In the first example it is not optimal to have constant growth rates, whatever the initial values of the stock of capital and the stock of pollution are. This is in sharp contrast with the standard Rebelo model without pollution. Therefore, in this case no steady state analysis is warranted. On the other hand, along an optimal program the growth rates will approach constants. In the second example the steady state is optimal for a given initial stock constellation, and a comparative statics analysis on the steady state is warranted.

5. Conclusions

It has been shown in this paper that negative environmental externalities may affect optimal long run growth rates and that restricting attention to balanced growth is not always warranted. The main conclusion to be drawn from this seems to be that stock effects should indeed be incorporated in optimal growth models, not only because this is important from an economic point of view, but also because this may lead to outcomes which differ drastically from the ones which arise when only the negative flow effect is taken into account.

References

Barro, J. (1990), 'Government Spending in a Simple Model of Endogenous Growth', *Journal of Political Economy* **98**(5), S103–S125.

Bovenberg, A. and S. Smulders (1993), 'Environmental Quality and Pollution Saving Technological Change: A Two-Sector Endogenous Growth Model', mimeo, Tilburg University.

Brock, W. and J. Scheinkman (1976), 'Global Asymptotic Stability of Optimal Control Systems with Applications to the Theory of Economic Growth', *Journal of Economic Theory* **12**, 164–190.

Forster, B. (1973), 'Optimal Capital Accumulation in a Polluted Environment', *Southern Economic Journal* **39**, 544–547.

Gottinger, H. (1993), Two essays in energy economics: long run investment and technical progress in dynamic and vintage models.

Gradus, R. and S. Smulders (1993), 'The Trade-off between Environmental Care and Long-Term Growth-Pollution in Three Prototype Growth Models', *Journal of Economics* **58**, 25–51.

Hofkes, M. (1993), 'Some Notes on the Concept of Sustainable Development in Endogenous Growth Models', mimeo, Free University, Amsterdam.

Huijberts, H. and C. Withagen (1992), 'Local Asymptotic Stability of Optimal Steady States', *Economics Letters* **40**, 7–11.

Keeler, E., M. Spencer, and R. Zeckhauser (1971), 'The Optimal Control of Pollution', *Journal of Economic Theory* **4**, 19–34.

Lucas, R. E. (1988), 'On the Mechanics of Economic Development', *Journal of Monetary Economics* **22**, 3–42.

Marrewijk, Ch., C. de Vries, and C. Withagen (1992), 'Optimal Localized Production Experience and Schooling', *International Economic Review* **33**, 91–110.

Michel, P. and G. Rotillon (1993), 'Pollution's Disutility and Endogenous Growth', mimeo, Université de Paris.

Ploeg, F. v.d. and C. Withagen (1991), 'Pollution Control and the Ramsey Problem', *Environmental and Resource Economics* **1**, 215–236.

Ploeg, F. van der, J. Verbeek, and C. van Marrewijk (1993), 'Pollution, Abatement and Endogenous Growth', mimeo, Erasmus University, Rotterdam.

Rebelo, S. (1991), 'Long Run Policy Analysis and Long Run Growth', *Journal of Political Economy* **99**, 500–521.

Romer, P. M. (1986), 'Increasing Returns and Long Run Growth', *Journal of Political Economy* **94**, 1002–1037.

Smulders, S. and R. Gradus (1993), 'Pollution Abatement and Long-Term Growth', mimeo, Tilburg University.

Vellinga, N. (1993), 'Endogenous Growth and the Environment: Increasing Returns to Scale and Human Capital', mimeo, Eindhoven University of Technology.